

SUMMARY OF THE CONIC SECTIONS

GEOMETRIC DEFINITION: 'Conic sections' come from slicing the cone at different angles:

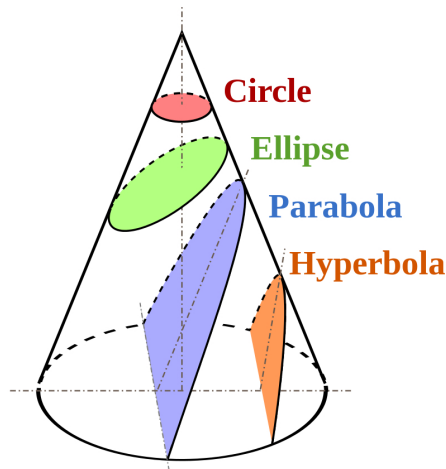


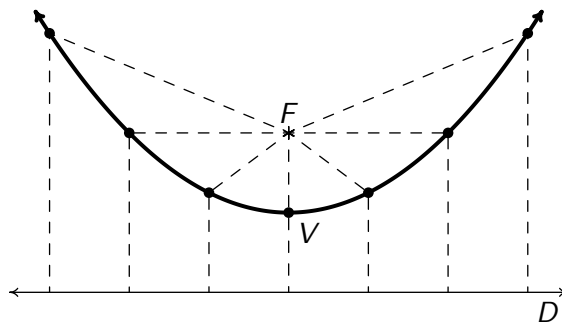
Image taken from [Wikipedia](#).

They can also be described as a **locus** (set) of points in the plane which satisfy certain distance conditions.

ALGEBRAIC DEFINITION: The conic sections arise from graphing quadratic equations in two variables:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

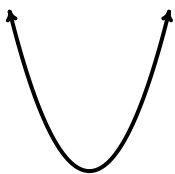
PARABOLAS: Given a fixed line called the **directrix**, D , and a fixed point not on the line called the **focus**, F , a **parabola** is the set of all points whose distance to the directrix is the same as their distance to the focus.



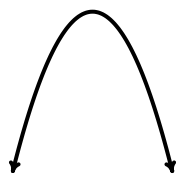
Equations of parabolas have only **one** squared term, and there are two standard forms:

$$(x - h)^2 = 4p(y - k)$$

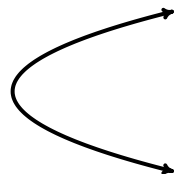
$$(y - k)^2 = 4p(x - h)$$



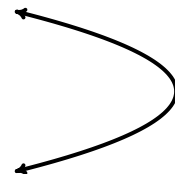
$$p > 0$$



$$p < 0$$



$$p > 0$$

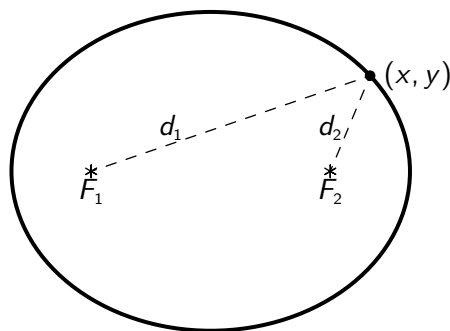
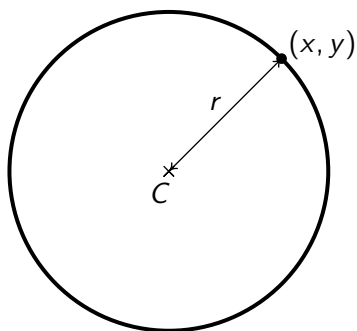


$$p < 0$$

In each case:

- the vertex is (h, k) .
- p is the *directed* distance from the vertex to the focus (and from the vertex to the directrix.)
- $|4p|$ is the width of the parabola at the focus (the length of the latus rectum)

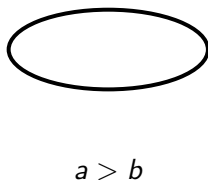
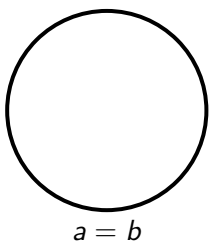
CIRCLES AND ELLIPSES: Given a fixed point called the **center**, C , and a fixed distance called the **radius**, r , a **circle** is the set of all points r units away from the center. Given two fixed points called **foci**, F_1 and F_2 , and a fixed distance d , an **ellipse** is the set of all points the **sum** of whose distances to the foci is d .



$$d_1 + d_2 = d \text{ for all } (x, y) \text{ on the ellipse}$$

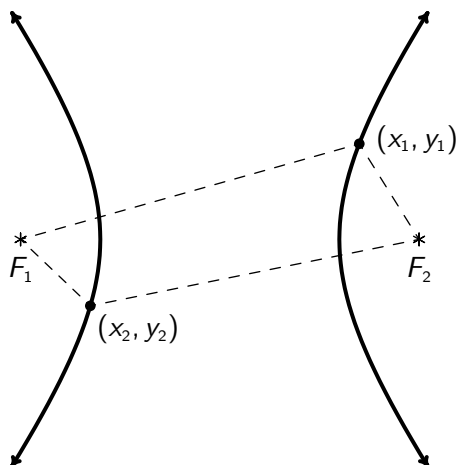
Equations of circles and ellipses contain **two** squared terms with the **same** signs. Equal coefficients point to a circle; unequal coefficients indicate an ellipse. The standard form is:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$



- The center in each case is (h, k) .
- 'a' tells us how far to move in x-direction from the center (i.e., the horizontal stretch.)
- 'b' tells us how far to move in the y-direction from the center. (i.e., the vertical stretch.)
- If $a = b$, the curve is a circle, and $a = b = r$ is the radius of the circle.
- If $a > b$, the curve is an ellipse with a horizontal major axis.
 - The vertices are 'a' units to the left and right of the center.
 - The foci are $c = \sqrt{a^2 - b^2}$ units to the left and right of the center.
- If $b > a$, the curve is an ellipse with a vertical major axis.
 - The vertices are 'b' units above and below the center.
 - The foci are $c = \sqrt{b^2 - a^2}$ units above and below the center.
- The eccentricity, $e = \frac{\text{distance from center to focus}}{\text{distance from center to vertex}} = \frac{c}{a \text{ or } b, \text{ whichever is bigger}}$
 - e is a measure of 'roundness' and for ellipses, $0 < e < 1$.
 - If $e \approx 0$, the ellipse is more circular; if $e \approx 1$, the ellipse is less circular (more 'eccentric'.)

HYPERBOLAS: Given two fixed points called **foci**, F_1 and F_2 , and a fixed distance d , a **hyperbola** is the set of all points the **difference** of whose distances to the foci is d .



In the figure above:

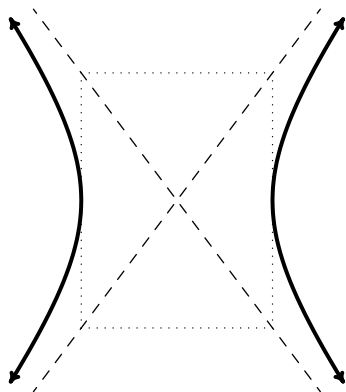
the distance from F_1 to (x_1, y_1) – the distance from F_2 to (x_1, y_1) = d

and

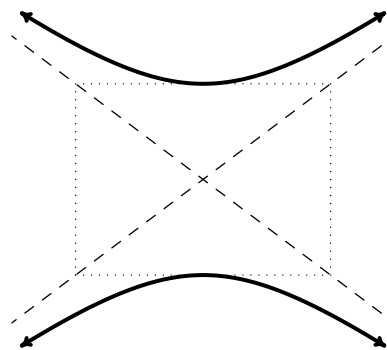
the distance from F_2 to (x_2, y_2) – the distance from F_1 to (x_2, y_2) = d

Equations of hyperbolas contain **two** squared terms with **different** signs. There are two standard forms:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$



- The center in each case is (h, k) .
- To sketch a 'guide rectangle:' from the center, move ' a ' units left and right and ' b ' units up and down.
- The slant asymptotes contain the center, (h, k) and have slopes $\pm b/a$.
- If the standard form contains $x^2 - y^2$, the curve is a hyperbola with a horizontal transverse axis.
 - The vertices are ' a ' units to the left and right of the center.
 - The foci are $c = \sqrt{a^2 + b^2}$ units to the left and right of the center.
- If the standard form contains $y^2 - x^2$, the curve is a hyperbola with a vertical transverse axis.
 - The vertices are ' b ' units above and below the center.
 - The foci are $c = \sqrt{b^2 + a^2}$ units above and below the center.